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Trigonometric Identities

Pythagorean Identities

$\sin^2\alpha + \cos^2\alpha = 1$ $1 + \cot^2\alpha = \csc^2\alpha$ $1 + \tan^2\alpha = \sec^2\alpha$

Symmetry or Sign Identities

Odd functions: $\sin(-x) = -\sin x$ $\tan(-x) = -\tan x$ $\csc(-x) = -\csc x$ $\cot(-x) = -\cot x$

Even functions: $\cos(-x) = \cos x$ $\sec(-x) = \sec x$

Cofunction or Complement Identities

$\sin(\pi/2 - \theta) = \cos\theta$ $\cos(\pi/2 - \theta) = \sin\theta$ $\tan(\pi/2 - \theta) = \cot\theta$

$\sin(\theta - \pi/2) = -\cos\theta$ $\cos(\theta - \pi/2) = \sin\theta$ $\tan(\theta - \pi/2) = -\cot\theta$

$\sin(\theta + \pi/2) = \cos\theta$ $\cos(\theta + \pi/2) = -\sin\theta$ $\tan(\theta + \pi/2) = -\cot\theta$

Supplement Identities

$\sin(\pi - \theta) = \sin\theta$ $\cos(\pi - \theta) = -\cos\theta$ $\tan(\pi - \theta) = -\tan\theta$

$\sin(\theta - \pi) = -\sin\theta$ $\cos(\theta - \pi) = -\cos\theta$ $\tan(\theta - \pi) = \tan\theta$

$\sin(\pi + \theta) = -\sin\theta$ $\cos(\pi + \theta) = -\cos\theta$ $\tan(\pi + \theta) = \tan\theta$

Sum and Difference Angles Identities

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Double-Angle Identities

$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$ $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Half-Angle Identities

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1 - \cos\theta}{2}}$ $\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1 + \cos\theta}{2}}$ $\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1 - \cos\theta}{1 + \cos\theta}} = \dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{1 - \cos\theta}{\sin\theta}$

Product-to-Sum Identities

$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$

$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$

$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$

$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]$

Sum-to-Product Identities

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

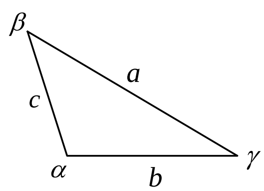
$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\cos B + \cos A = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\cos B - \cos A = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos\gamma$



Heron's Formula

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$s = (a + b + c)/2$

Law of Sines

$\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$

Area $= \dfrac{1}{2}\cdot b\cdot a\sin\gamma = \dfrac{1}{2}\cdot base\cdot altitude$

Area of a triangle equals one half the product of two sides and the sine of the included angle.

SSA (ASS) Ambiguity

If the side opposite the angle is less than the side adjacent to the angle it is possible to have one, two or no triangles formed: If you know only γ, a and c, and $c < a$, you may have two triangles, one with β obtuse and one with it acute; or you may have one triangle if $\alpha = 90^\circ$; or no triangle if c is too short to reach b (an invalid trigonometric domain value will occur)