Triangles

Triangle Names

Triangles can be characterized by the measure of their sides or their angles:

Angles

- **Acute**: All angles are less than 90°.
- **Right**: One angle is 90°.
- **Obtuse**: One angle is greater than 90°.
- **Oblique**: Not a right triangle. This means it is either Acute or Obtuse.
- **Equiangular**: All three angles are equal. This also means it is equilateral, see below.

Sides

- **Scalene**: No sides are of equal length.
- **Isosceles**: At least two sides have equal length.
- **Equilateral**: All three sides have equal length. This also means it is isosceles and equiangular.

Congruent Triangles

Triangles that have the following combinations of congruent corresponding parts are congruent.

- Three sides are congruent (SSS)
- Any two angles and a side are congruent (ASA and AAS)
- Two sides and their included angle are congruent (SAS).

Importantly the SSA (ASS) case *does not* prove congruency. This is called the "ambiguous case" because depending on the side lengths it is possible to construct two different triangles.

Similar Triangles

Triangles that have the following combinations of congruent corresponding angles and *proportional* corresponding sides are similar.

- All three sides are in the same *proportion* (SSS).
- Any two angles are congruent (AA). The third angle is then congruent since the angles must sum to 180°.
- Two sides are in the same *proportion* and their included angle is congruent (SAS).

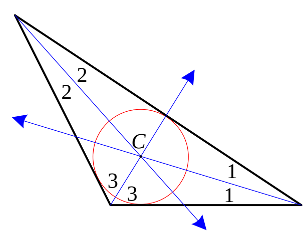
Triangle Centers

There are many different definitions for centers of a triangle. The four most common are defined below. See the website https://www.mathopenref.com/trianglecircumcenter.html to graphically explore these centers.

Each of these triangle centers are formed by the intersection of three lines. Such a point is called a point of concurrency.

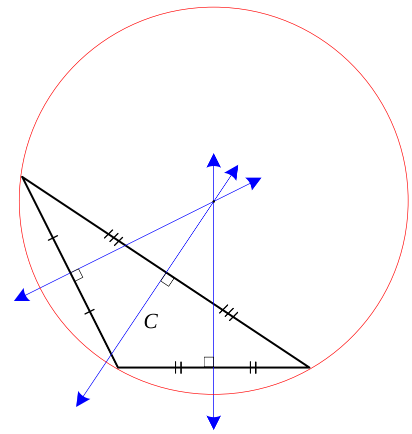
Incenter

- Is the point of concurrency of the triangle's three *angle bisectors*.
- Is always in the interior of the triangle.
- Is the center of the largest circle that will fit within the interior of the triangle. This circle is called the incircle or inscribed circle.
- The sides of the triangle are tangent to the incircle.



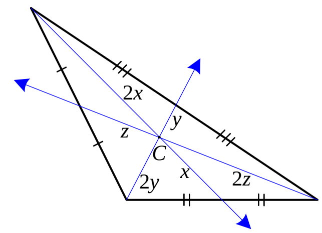
Circumcenter

- Is the point of concurrency of the lines collinear with the three *perpendicular bisectors* of the triangle's sides.
- Lies in the interior of an acute triangle, lies in the exterior of an obtuse triangle, and lies on the midpoint of the hypotenuse of a right triangle.
- Is the center of the circle that intersects the three vertices of the triangle. This circle is called the circumcircle.



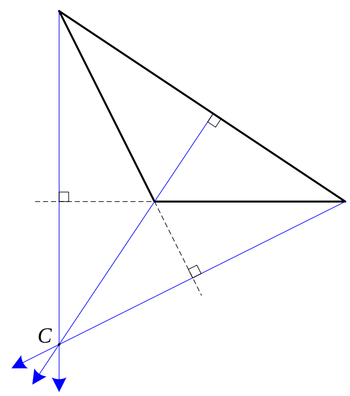
Centroid

- Is the point of concurrency of the three *medians* of the triangle.
- Is always in the interior of the triangle.
- Is the "center of mass": a uniformly dense triangular object will balance at its centroid.
- The centroid splits each median so that the segment from the vertex to the centroid is twice the length of the segment from the centroid to the side. Put another way, the centroid is 2/3 of the way along the median measured from the vertex and 1/3 as measured from the side.
- Each median also splits the given triangle into two parts of equal area.



Orthocenter

- Is the point of concurrency of the lines collinear with the three *altitudes* of the triangle.
- Is in the interior of an acute triangle, exterior of an obtuse triangle, and lies on the vertex of the right angle of a right triangle.



Euler Line

There is a special relationship between the last three centers above: The circumcenter, centroid and orthocenter are all collinear. The line they form is called the **Euler line**.

In an equilateral triangle all four centers are the same point since the corresponding altitudes, medians, perpendicular bisectors, and angle bisectors of an equilateral triangle are collinear. In this case the Euler line degenerates to a point.