

## Axioms and Properties of Real Numbers

All variables and numbers on this page represent real numbers.

### Axioms of Real Numbers

Field Axioms	Addition	Multiplication
Closure	The sum of real numbers is a real number.	The product of real numbers is a real number.
Commutative	$a + b = b + a.$	$a \cdot b = b \cdot a.$
Associative	$(a + b) + c = a + (b + c).$	$(a \cdot b) \cdot c = a \cdot (b \cdot c).$
Distributive	Multiplication distributes over addition: $c(a + b) = ca + cb.$	
Identity	0 is the additive identity. $a + 0 = a.$	1 is the multiplicative identity. $a \cdot 1 = a$
Inverses	A number and its negative are additive inverses. $a + (-a) = 0.$	A number and its reciprocal are multiplicative inverses $a \cdot \frac{1}{a} = 1, a \neq 0.$

### Axioms of Equality

Reflexive	$a = a$
Symmetric	If $a = b$ , then $b = a$
Transitive	If $a = b$ and $b = c$ then $a = c.$

### Axioms of Order

Additive	If $a > b$ , then $a + c > b + c.$
Transitive	If $a > b$ and $b > c$ then $a > c.$
Trichotomy	Either $a > b$ or $a = b$ or $a < c.$
Positive Multiplication	If $c > 0$ and $a > b$ then $a \cdot c > b \cdot c$
Negative Multiplication	If $c < 0$ and $a > b$ then $a \cdot c < b \cdot c$

### Definition of Subtraction and Division

Subtraction	$a - b = a + (-b).$ Subtraction and addition are inverse operations.
Division	For $b \neq 0$ , $a \div b = \frac{a}{b} = a \cdot \frac{1}{b}.$ Division and multiplication are inverses operations.

### Properties of Real Numbers

The properties of real numbers below can be proven given the axioms above.

#### Commonly Used Properties

Uniqueness of identity elements	The additive identity 0, and the multiplicative identity 1, are unique.
Uniqueness of negative and reciprocal	A number's negative and a non-zero number's reciprocal are unique.
Multiplication Property of 0	$a \cdot 0 = 0.$
Multiplication Property of -1	$-1 \cdot a = -a$
Addition Property of Equality	If $a = b$ then $a + c = b + c.$
Subtraction Equality Property	If $a = b$ then $a - c = b - c$
Multiplication Property of Equality	If $a = b$ then $a \cdot c = b \cdot c.$
Division Property of Equality	If $a = b$ and $c \neq 0$ then $a/c = b/c$

#### Properties of Order

Reciprocal	If both sides of an inequality have the same sign, then the sense of the inequality is reversed by taking the reciprocal of each side.
Symmetric	If $a < b$ , then $b > a.$ If $a \leq b$ , then $b \geq a.$
Transitive Extension	In the transitive axiom $>$ can be replace with $\leq, <, \text{ or } \geq.$