Probability

A **permutation** is a distinct sequence of a subset. Being a sequence, *the elements and their order* distinguish different permutations.

A **combination** is a distinct subset. Being a set, *the elements and not their order* distinguish different combinations.

Given a set of *n* elements from which *r* are chosen (without replacement) there are ${}_nP_r$ permutations and ${}_nC_r$ combinations where

$$ {}_nP_r = \frac{n!}{(n-r)!} \quad \text{and} \quad {}_nC_r = \frac{n!}{(n-r)!r!} = \frac{{}_nP_r}{r!} . $$

A **random experiment** is an experiment where the result or observation, is not deterministic. That is, it may result in different observations when repeated.

A single result or observation of a random experiment is called an **outcome**. Some texts require outcomes to be defined so they are *equally likely results* of a random experiment.

The **sample space** of an experiment is the set of all possible outcomes of the experiment.

An **event** is a subset of the sample space or equivalently a set of outcomes. Different events may have different likelihoods of occurring depending on the subset of the sample set chosen.

The notation $n(E)$ indicates the number of possible outcomes of a random experiment in which the event E occurs. If S is the sample set and *if each outcome is equally likely*, then the probability of event E occurring is $P(E) = n(E) / n(S)$. This is the **classic probability rule** which requires equally likely outcomes.

If a random experiment is repeated n times and $f(E)$ represents the number of times event E occurs in those n trials, then $\lim_{n\to\infty} \frac{f(E)}{n} = P(E)$. This is the **relative frequency probability rule** and it applicable regardless of the likelihood of each outcome.

The **conditional probability** of event B occurring given event A has already occurred is written as $P(B \mid A)$. Since both events A and B must occur, we begin with $P(B \cap A)$, but since A must have occurred, we "normalize" this by dividing by $P(A)$, effectively reducing the sample space to A. This gives

$$ P(B \mid A) = \frac{P(B \cap A)}{P(A)} . $$

The probability of the events ***A or B*** occurring is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

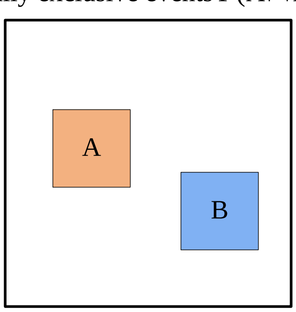
- If A and B are *mutually exclusive* this means $P(A \cap B) = 0 \;\rightarrow\; P(A \cup B) = P(A) + P(B)$.

The probability of the events ***A and B*** both occurring is $P(A \cap B) = P(A) \cdot P(B \mid A)$

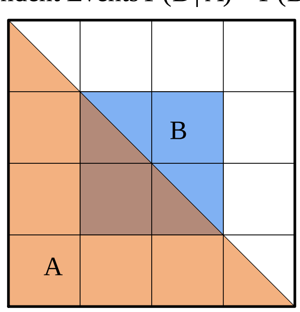
- If A and B are *independent* events, then $P(B \mid A) = P(B) \;\rightarrow\; P(A \cap B) = P(A) \cdot P(B)$.

Independent events cannot be mutually exclusive and vice versa (excluding zero probabilities). In a sense they are opposites: If one of two mutually exclusive events occur this completely determines the probability of the other – it is zero; if one of two independent events occur this has no influence on the probability of the other.

Mutually exclusive events $P(A \cap B) = 0$



Independent Events $P(B \mid A) = P(B) = 1/4$



If a sample space can be divided into two where a person has either "a condition", called a positive condition ($C+$), or does not, called a negative condition ($C-$), and the same sample space can also be divided into two where a "test for the condition" is either positive, called a positive test ($T+$), or negative, called a negative test ($T-$), then we define the following terms:

- **Specificity** = $P(T-|C-)=\dfrac{P(T- \cap C-)}{P(C-)}$. This is also called true negative rate.
- **Sensitivity** = $P(T+|C+)=\dfrac{P(T+ \cap C+)}{P(C+)}$. This is also called true positive rate.