www.cmtutoring.com cameron@cmtutoring.com Cameron McClintock

Factoring Quadratics

An unfactored quadratic in x looks like $ax^2 - bx - c$. To factor it we need to separate the x coefficient into two terms such that their sum is still b while their product is ac. This is the real work in factoring. The techniques shown in the following sections, except for the roots method, assume you have done this and are means to assemble the two factors of the quadratic from these two numbers.

Before breaking up the x term first factor out any common factors and if the coefficient of the squared term is negative factor out -1.

The techniques below are illustrated with the example $-18x^2 + 57x + 21 = -3(6x^2 - 19x - 7)$. We first deal with the quadratic in the parentheses and then re-introduce the common factor at the end. The split coefficients of x in this example are 2 and -21 since they sum to -19 and multiply to -42.

The last section describes the discriminant which is used to test if the roots of a quadratic equation are rational, irrational, or complex.

Grouping

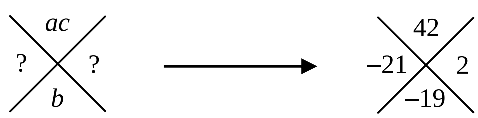
This uses straight forward algebra without any of the visual aids/distractions of the diamond and box methods. Once the two coefficients of x are identified, separate the quadratic into two groups and factor out common terms,

$$\begin{aligned} 6x^2 - 19x - 7 &= 6x^2 + 2x - 21x - 7 \\ &= \left(6x^2 + 2x\right) - \left(21x + 7\right) \\ &= 2x\left(3x+1\right) - 7\left(3x+1\right) \\ &= \left(3x+1\right)\left(2x-7\right). \end{aligned}$$

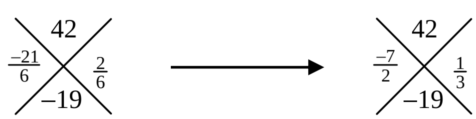
Finally re-introduce the common factor to give $-18x^2 + 57x + 21 = -3\left(3x+1\right)\left(2x-7\right)$.

Diamond Method

Draw a cross pattern as show below (not sure why it is not called the cross method), and fill in the top area with the product ac and the bottom area with b. The two coefficients of the x term go on the left and right,



Now divide the left and right numbers by a and then simplify the fractions.



Now the factors can be formed from the left and right terms: Keep any negative sign associated with the numerator; the x coefficient is the denominator; the constant term is the numerator. This gives $(2x-7)(3x+1)$.

Finally re-introduce the common factor to give $-18x^2 + 57x + 21 = -3\left(3x+1\right)\left(2x-7\right)$.

Box Method

Draw a box divided into four as show below. Fill in the top left with ax^2 and the bottom right with c. The separated x terms go in the remaining two cells.

Write down common factors of each row in a new column to the left; write down common factors of each column in a new row above. In each case retain the sign of the term nearest the edge.

ax^2	$?\,x$
$?\,x$	c

$\longrightarrow$

	$3x$	1
$2x$	$6x^2$	$2x$
-7	$-21x$	-7

The newly written column forms one factor and the newly written row forms the other, $(2x-7)(3x+1)$.

Finally re-introduce the common factor to give $-18x^2+57x+21=-3(3x+1)(2x-7)$.

Roots Method

Find the roots of the quadratic equation $P(x)=ax^2-bx-c=0$ using either the quadratic formula or by completing the square. If the roots are d and e then the factors of $P(x)$ are $a(x-d)(x-e)$. If possible, distribute a to give integer terms within the parentheses. A different example, $192x^2+512x+285$, is used in this case. It looks onerous to find how to divide up the x term so rather than attempt this we use the roots method.

$$\begin{aligned} x &= \frac{-512 \pm \sqrt{512^2-4(192)(285)}}{2(192)} \\ &= \frac{-512 \pm 208}{384} \\ &= -\frac{19}{24} \text{ and } -\frac{15}{8}. \end{aligned}$$

The factors can then be formed as $192(x+19/24)(x+15/8)$. Distributing the common factor so that the constant terms become integers give: $192x^2+512x+285=(24x+19)(8x+15)$.

Discriminant Test

The discriminant of a quadratic equation $ax^2-bx-c=0$ is often denoted as D and is defined as $D \triangleq b^2-4ac$.

If the quadratic has *real coefficients* the value of the discriminant tells you the following about the quadratic's roots:

- $D > 0$ means both roots are real and distinct. Further, if the *coefficients* are *rational* and
  - D is a perfect square, then the roots are *rational*.
  - D is not a perfect square, then the roots are irrational and the quadratic cannot be factored into products with rational coefficients and is said to be *prime* or *irreducible* (over the rationals).
- $D = 0$ means the roots are real and duplicate (multiplicity 2). If the coefficients are rational so are the roots.
- $D < 0$ means the roots are complex conjugates.