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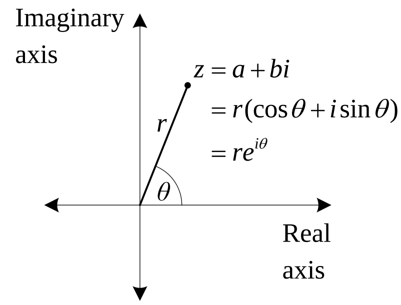
Complex Numbers

Complex Number Notation

Rectangular Notation	$z = a + bi$	a is the real part; b is the imaginary part. A point is represented on a rectangular coordinate system as (a,b).
Polar (also called Trigonometric) Notation	$z = r(\cos\theta + i\sin\theta)$	r is the magnitude and θ is the argument. A point is represented on a polar coordinate system as (r,θ).
Exponential Notation	$z = re^{i\theta}$	



$r = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1} b/a$ *

*adjusted to the correct quadrant by examining the sign of a and b

- The **argument** θ is the angle formed from the positive real axis to the segment with endpoints at the origin and point (a,b).
- $\theta = \tan^{-1} b/a$, adjusted to the correct quadrant by examining the sign of a and b.
- The **absolute value**, or **magnitude** or **modulus** of z is denoted as $|z| = r = \sqrt{a^2 + b^2}$.
- The **real** and **imaginary** part of $z = a + bi$ can be written as $\operatorname{Re} z = a$ and $\operatorname{Im} z = b$ respectively.
- A **purely imaginary** complex number has $\operatorname{Re} z = 0$ and $\operatorname{Im} z \neq 0$.
- The **complex conjugate** of $z = a + bi$ is $a - bi$ and is denoted $\bar{z}$. Also, $|z|^2 = z\bar{z}$.

Operations on Complex Numbers

Operation	Rectangular $z = a + bi$	Polar $z = r(\cos\theta + i\sin\theta)$	Exponential $z = re^{i\theta}$
Addition/Subtraction	$z_1 \pm z_2 = (a_1 \pm a_2) + (b_1 \pm b_2)i$	Convert to rectangular	Convert to rectangular
Multiplication	$z_1 \cdot z_2 = (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i$	$z_1z_2 = r_1r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$	$z_1z_2 = r_1r_2e^{i(\theta_1+\theta_2)}$
Division	$\frac{z_1}{z_2} = \frac{z_1}{z_2}\frac{\bar{z}_2}{\bar{z}_2} = \frac{1}{\|z_2\|^2} z_1\bar{z}_2$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2))$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}e^{i(\theta_1-\theta_2)}$
n^{th} Power DeMoirve's Theorem	$z^n = zz\cdots z$, n times; Convert to polar/exponential for larger n.	$z^n = r^n(\cos n\theta + i\sin n\theta)$	$z^n = r^ne^{in\theta}$
n^{th} Root	Convert to polar/exponential	$z^{1/n} = r^{1/n}\left(\cos\left(\frac{\theta}{n} + k\frac{2\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + k\frac{2\pi}{n}\right)\right)$ $k = 0,1,2\cdots n-1.$	$z^{1/n} = r^{1/n}e^{i\theta/n + k2\pi/n}$ $k = 0,1,2\cdots n-1.$

Converting between Polar/Exponential and Rectangular

Given $a + bi$ to convert to polar/exponential form find the argument θ and magnitude r:

- $\theta = \tan^{-1} b/a$, adjusted to the correct quadrant by examining the sign of a and b.
- $r = \sqrt{a^2 + b^2}$

Given the argument θ and magnitude r to convert to rectangular form find the real a and imaginary b components:

- Determine the sign of a and b from the quadrant of θ.
- Determine the magnitude of a and b: $|a| = r\cos\theta$, $|b| = r\sin\theta$.